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A new lattice measurement for potentials between static SU(3) sources

S. Deldar[a]

Department of Physics, University of Tehran, P.O. Box 14395/547, Tehran 1439955961, Iran

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Abstract. In this article, a new calculation of the static potentials between sources of different representations in $SU(3)$ gauge group is presented. The results of the author's previous study [Phys. Rev. D **62**, 034 509 (2000)] at the smallest lattice spacing $a_s \simeq 0.11$ fm are shown to have been affected by finite volume effects. Within statistical errors, the new results obtained here are still in agreement with both Casimir scaling and flux tube counting. There is also no contradiction to the results obtained by Bali [Phys. Rev. D **62**, 114 503 (2000)] which however exclude flux counting.

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1 Introduction

Studying the potential between static sources in QCD is still an interesting subject. At large distances, linearity of the potential leads to the confinement which is one of the most challenging issues in QCD. There are various confinement hypotheses. The Casimir scaling, sine scaling and flux tube counting are among those hypotheses which try to describe the potential between static sources in different representations. The question of the Casimir scaling/sine scaling/flux tube counting is essential for our understanding of the confinement mechanism and to clarify the correspondence between quantum field theories at a large number of colors and string theories on certain compactified manifolds.

At short distances, potentials between two quarks can be calculated using perturbation theory. For this regime, called asymptotic freedom, the coupling constant is small enough to use perturbation theory. As the distance between quarks increases, a tube of chromoelectric flux is formed between them and a potential proportional to the quark separation is expected. Perturbation theory dose not work for this region. Lattice gauge theory is one of the most successful method which works well for this low energy regime where the potential may be defined by

$$V(r) \simeq A/r + Kr + C, \tag{1}$$

where r and K indicate the quark separation and the string tension, respectively. The first term shows the Coulombic potential which is the result of one gluon exchange at short distances. The confinement is understood from the second term which shows that the potential between the pair of quark–antiquark increases by distance. Quite a few lattice studies have been devoted to the calculations of string tensions in the closed string sector (torelons) [3], yet only two authors, Deldar [1] and Bali [2], have recently systematically studied the situation in the open string sector and have measured the potentials between static sources for a variety of representations. Both calculations have shown good agreement with Casimir scaling, especially based on Bali's paper; within an accuracy of 5 percent, no violation to the Casimir scaling is detected. Here, Casimir scaling means that the string tension for each representation is to be roughly proportional to the eigenvalue of the quadratic Casimir operator in that representation. The Casimir scaling regime is expected to exist for intermediate distances, perhaps extending from the onset of confinement to the onset of screening [4]. There is another argument on the linear part of the potential at intermediate distances which claims that the string tension at this region is proportional to the number of fundamental flux tubes embedded into the representation [5]. The fundamental flux or string is the one that connects a fundamental heavy quark with an antiquark. This idea is called "flux tube counting". In general, at very large N strings do not interact; therefore, one would expect that the tension of meta-stable strings is proportional to the number of flux tubes. Of course, there are $\frac{1}{N}$ corrections. And then, if one waits for long enough, these meta-stable strings will decay into the stable strings with the given N-ality, whose tension is likely to be described by the sine formula or by Casimir scaling in some approximation which is not yet known so far.

The confining string with N-ality k is usually called a k-string and σ_k is the corresponding string tension. For $SU(N)$ gauge sources at large distances, where the strings

[a] e-mail: sdeldar@ut.ac.ir

are stable and do not decay, there exist some different theories about the ratio of $(\frac{\sigma_k}{\sigma f})$ where σf and σ_k are the string tensions of the fundamental quarks and k sources, respectively. The most trivial idea is to assume that the total flux is carried by k independent fundamental tubes:

$$\sigma_k = k\sigma_f. \tag{2}$$

Because of charge conjugation, $\sigma_k = \sigma_{N-k}$. Thus, for the $SU(3)$ gauge group, $\sigma_2 = \sigma_1$ and one universal string tension is obtained. In this case, string tensions of non-zero triality representations will be equal to the string tension of fundamental quarks at large distances. Asymptotic Casimir scaling is another theory about k-string ratios [6]:

$$\frac{\sigma_k}{\sigma f} = \frac{k(N-k)}{N-1}. \tag{3}$$

Another hypothesis is based on calculations in brane M-theory [7] or sine-law scaling:

$$\frac{\sigma_k}{\sigma_f} = \frac{\sin\frac{k\pi}{N}}{\sin\frac{\pi}{N}}. \tag{4}$$

In the large N limit, Casimir and sine-law scaling will be the same and equal to k.

Back to $SU(3)$ gauge group: at large distances, larger than 1.2 fm, where the potential between two sources is large enough, a pair of adjoint sources is released from the vacuum and string breaking may happen. Then, based on the triality of the representation, we expect to see screening or a change of the slope of the potential to the slope of the potential of the fundamental representation. Since there is only one independent string tension for an $SU(3)$ gauge field which is the string tension of the fundamental representation, the potentials of zero triality representations like the adjoint representation are expected to be screened at large enough distances, which means that the string tensions will be equal to zero, and for non-zero triality representations, one expects to see the string tension of the fundamental representation.

Although in this study potentials for distances larger than 1.2 fm are calculated, no string breaking and therefore no sign of the screening or the change of the slope of the potentials to that of the fundamental representation is observed. As mentioned in the previous paper [1], this is probably because of the fact that the Wilson loops do not couple well to the screened representations. As will be explained in the next section, the potentials are calculated by measuring the Wilson loops. Therefore, through this article, the intermediate string tensions and two of the theories which may be applied to this region are discussed. These theories are flux tube counting and the Casimir scaling for intermediate distances. Equation (2)–(4) for stable strings, which may be applied to large distances, are not investigated. I should mention that no comparison with MQCD or sine scaling could be done, since as far as the author knows, no calculations in that framework have been done for meta-stable strings[1]. There are comparisons with both Casimir scaling and sine scaling in the studies which measure the string tensions in the closed string sector (torelons) [3].

In spite of the good agreement between the author's previous calculations and Casimir scaling, the results of one of the lattices have not been scaled well with the others. The simulations were done on a couple of anisotropic lattices with spatial lattice spacings of 0.43, 0.25 and 0.11 fm. The results for lattice spacing 0.43 fm and 0.25 fm have been in good agreement, but the data for the finest lattice have not been scaled well with others, especially for higher representations. A new lattice spacing has been examined to probe the disagreement. This measurement is really important since in the author's previous calculations, the non-scaled potentials were the ones belonging to the finer lattice. In fact, in order to get the continuum, one has to use finer lattices; therefore, the results of the finer lattice should be more reliable. This is contrary to the author's previous calculations, where the finer lattice has not behaved properly. In this paper, it will be shown that the problem with the finer lattice has been the significantly smaller volume compared with the others, which leads to the finite volume effect error that is one of the most important errors of lattice gauge theory calculations. On the other hand, with the new lattice spacing, $a_s = 0.19$ fm, the lattice is still considered fine, and the volume is large enough to not encounter the finite volume error.

The hypotheses of Casimir scaling and flux tube counting for the meta-stable strings are also investigated, and it will be shown that string tensions are roughly in agreement with both theories.

The paper is organized as follows: in section two the Wilson loops and the potentials and in section three the action and the lattice are discussed. Results, scaling behavior, the string tensions and their features and the conclusion are discussed in the next sections, respectively.

2 Wilson loops and the static potentials

The potential between two static quarks is found by measuring the Wilson loop and looking for the area law fall-off for large t:

$$W(r,t) \simeq \exp[-V(r)t]. \tag{5}$$

$W(r,t)$ is the Wilson loop as a function of r, the distance between two sources, and t, the propagation time. The potential at distance r is determined from the asymptotic behavior of the Wilson loop, $W(r,t)$:

$$V(r) \simeq \lim_{t\to+\infty} \ln\left(\frac{W(r,t)}{W(r,t+1)}\right). \tag{6}$$

The string tension and the coefficient of the Coulombic term may be obtained by fitting $V(r)$ to (1). The $V(r)$ of different r are obtained from (6). The Wilson loops of

[1] Many thanks to M. Shifman, M. Teper for answering my questions about MQCD or sine-scaling theory

higher representations R, W_R, are calculated from the fundamental Wilson loop, U, by the tensor product method. The trace of W_R for representations 6, 8, 10, 15 symmetric, for 15 antisymmetric and for 27, which are the representations studied in this paper, are as follows:

$$\mathrm{tr}(W_6) = 1/2\left[(\mathrm{tr}U)^2 + \mathrm{tr}U^2\right], \quad (7)$$

$$\mathrm{tr}(W_8) = \mathrm{tr}U^\star \mathrm{tr}U - 1, \quad (8)$$

$$\mathrm{tr}(W_{10}) = 1/6\left[(\mathrm{tr}U)^3 + 2\left(\mathrm{tr}U^3\right) + 3\mathrm{tr}U\mathrm{tr}U^2\right], \quad (9)$$

$$\mathrm{tr}(W_{15s}) = 1/24\left[(\mathrm{tr}U)^4 + 6(\mathrm{tr}U)^2\mathrm{tr}U^2 + 8\mathrm{tr}U\left(\mathrm{tr}U^3\right) + 3\left(\mathrm{tr}U^2\right)^2 + 6\mathrm{tr}U^4\right], \quad (10)$$

$$\mathrm{tr}(W_{15a}) = 1/2\mathrm{tr}U^\star\left[(\mathrm{tr}U)^2 + \mathrm{tr}U^2\right] - \mathrm{tr}U, \quad (11)$$

$$\mathrm{tr}(W_{27}) = 1/4\left[\mathrm{tr}U^2 + (\mathrm{tr}U)^2\right] \times \left[\left(\mathrm{tr}\,(U^\star)^2\right) + (\mathrm{tr}U^\star)^2\right] - \mathrm{tr}U\mathrm{tr}U^\star. \quad (12)$$

3 Action and the lattice

A $16^3 \times 24$ lattice has been used for this new measurement. The coupling constant is 2.7 and the ratio of the spatial lattice spacing to the temporal one, $\xi = \frac{a_s}{a_t}$, is equal to 2. The improved action used for this lattice is [8]

$$S = \beta\left\{\frac{5}{3}\frac{\Omega_{\mathrm{sp}}}{\xi u_s^4} + \frac{4}{3}\frac{\xi\Omega_{\mathrm{tp}}}{u_s^2 u_t^2} - \frac{1}{12}\frac{\Omega_{\mathrm{sr}}}{\xi u_s^6} - \frac{1}{12}\frac{\xi\Omega_{\mathrm{str}}}{u_s^4 u_s^2}\right\}, \quad (13)$$

where $\beta = 6/g^2$, g is the QCD coupling, and ξ is the aspect ratio ($\xi = a_s/a_t$ at tree level in perturbation theory). Ω_{sp} and Ω_{tp} include the sum over spatial and temporal plaquettes; Ω_{sr} and Ω_{str} include the sum over 2×1 spatial rectangular and short temporal rectangular (one temporal and two spatial links), respectively. For $a_t \ll a_s$ the discretization error of this action is $O(a_s^4, a_t^2, a_t a_s^2)$. The coefficients are determined at tadpole-improved tree level [9]. The spatial mean link, u_s, is given by

$$\left\langle \frac{1}{3}\mathrm{Retr}P_{ss'}\right\rangle^{\frac{1}{4}}, \quad (14)$$

where $P_{ss'}$ denotes the spatial plaquette. In general the temporal link u_t, can be determined from

$$u_t = \frac{\sqrt{\frac{1}{3}\left\langle \mathrm{Retr}P_{st}\right\rangle}}{u_s}, \quad (15)$$

where P_{st} is the spatial–temporal plaquette. When $a_t \ll a_s$, u_t, the temporal mean link can be fixed to $u_t = 1$, since its value in perturbation theory differs from unity by $O(\frac{a_t^2}{a_s^2})$. To minimize the excited state contamination in correlation functions, spatial links are smeared [10] (APE smearing).

4 Results

Using (6) and measuring the Wilson loops for a variety of t's for fixed r's, the potential $V(r)$ can be found. This process is repeated for several r's and the optimum $V(r)$ for each r is extracted from plots like Fig. 1. More details may be found in [1]. Figure 2 shows $V(r)$ versus r for all representations using the new coupling constant. The potentials have been fitted to (1) which has a linear plus a Coulombic term. The error bars on the points are the sum in quadratic of statistical and systematic errors. The systematic errors are due to the change of the fit range of V versus r. From the plot, one can see that the potentials are linear at intermediate distances and thus quarks are confined at this regime. At short distances, potentials are proportional to $\frac{1}{r}$. The slopes of the potentials of higher representations are expected to decrease at large distances so that zero triality representations are screened, and the slope of the potentials of quarks with non-zero triality representations changes to the slope of the potential between quarks in the fundamental representation. A change of the slope for the potential between gluons (quarks in the adjoint representation) is expected to happen at $r \simeq 1.2$ fm, where the potential is equal to the potential energy of the gluelumps. The gluon–antigluon released from the vacuum couple to the initial sources and screening would happen. For the coupling constant used in this calculation, which is $\beta = 2.7$, the lattice spatial spacing is about 0.19 fm and the maximum lattice distance is $r = 8$. Thus the maximum physical distance between static sources is about 1.5 fm, which is not really large enough to observe significant bending of screened potentials. In fact, even in the previous measurements where the maximum lattice distance was about 2.2 fm, this change of the slope of the potential or string

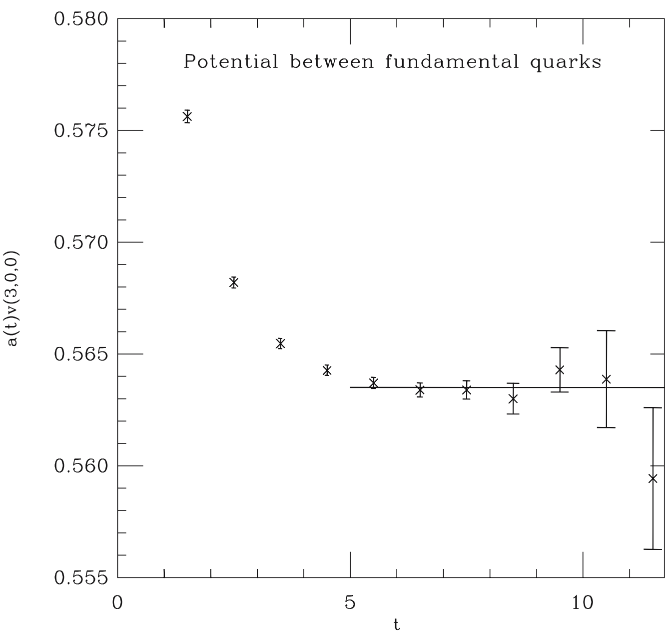

Fig. 1. Potential versus t for the fundamental representation. The fit range is shown by the *solid line*

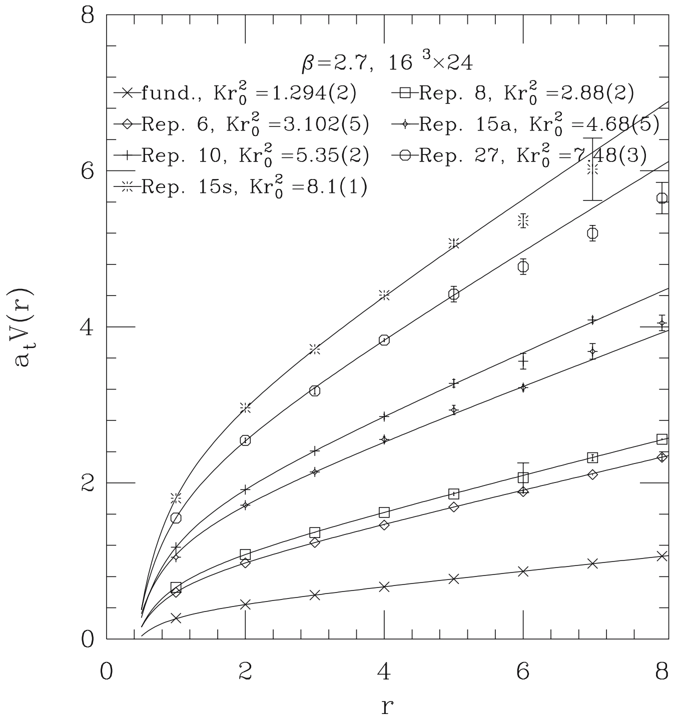

Fig. 2. Potentials for the fundamental, 6, 8, 10, 15a, 15s and 27 representations. The fits are based on 12800 measurements. Rough agreement with Casimir scaling is observed at the intermediate distances. Kr_0, the dimensionless string tension for each representation, is indicated in the plot. r_0 is the hadronic scale which is defined in terms of the force between static quarks at intermediate distance

breaking has not been observed. As discussed in [1], it is probably because of the well known fact that the Wilson loops do not couple well to screened states. It should be mentioned that the errors of the large distance potentials are large enough to not have a significant role in the fitting.

5 Scaling behavior

The actual measurement by lattice simulations is represented in lattice units. To convert the units into physical units, one has to look for a physical quantity with a known value. Then this reference quantity is measured on the lattice and by comparing the two values, the lattice spacing is extracted in physical units. The hadronic scale r_0, defined in terms of the force between static quarks at intermediate distance [11] can be used as the reference quantity:

$$[r^2 \mathrm{d}V/\mathrm{d}r]_{r=r_0} = 1.65, \tag{16}$$

where $V(r)$ is the static quark potential in the fundamental representation. The definition of (16) gives $r_0 \simeq 0.5\,\mathrm{fm}$ in a phenomenological potential model. The value $r_0^{-1} = 410 \pm 20\,\mathrm{MeV}$ as determined by Morningstar et al. [12] is used in this study. To set the scale and find a_s, I use (1) and the hadronic scale equation, (16):

$$\frac{r_0}{a_s} = \sqrt{\frac{1.65 - A}{K a_s^2}}. \tag{17}$$

For the anisotropic lattice, $K a_s a_t$ may be found from the fits. The input aspect ratio $\xi = \frac{a_s}{a_t} = 2$ has been chosen since the difference between the input value and lattice simulations is expected to vanish in the continuum limit. As shown by previous calculations, the good scaling behavior of the fundamental string tension is good evidence that this assumption is correct. It is now possible to show the results of different lattice simulations, using the scaled potentials and lattice distances in terms of the hadronic scale r_0. Figure 3 shows the static quark potential versus the hadronic scale r_0 for the following four lattices: $10^3 \times 24$, $18^3 \times 24$, $16^3 \times 24$, and $16^3 \times 24$ with spatial lattice spacings 0.45 fm, 0.25 fm, 0.11 fm and 0.19 fm, respectively. The new lattice is the one with lattice spacing equal to 0.19 fm. Proper scaling for all couplings for the fundamental representation is observed. In contrast to the fundamental representation where a good scaling is obtained for all four lattices, as the dimension of representation increases, the finest lattice, $a_s = 0.11\,\mathrm{fm}$, violates the scaling. Figure 4 shows the potential for representation 6. The finest lattice does not scale well, but the data for other three measurements are in good agreement. Figure 5, which shows the potentials for representations 15a, is another confirmation of this fact. The potentials of the other representations, 8, 10, 15s and 27, show the same behavior. In general, one expects to get the continuum by making the lattice spacing finer. However, as Figs. 4 and 5 indicate,

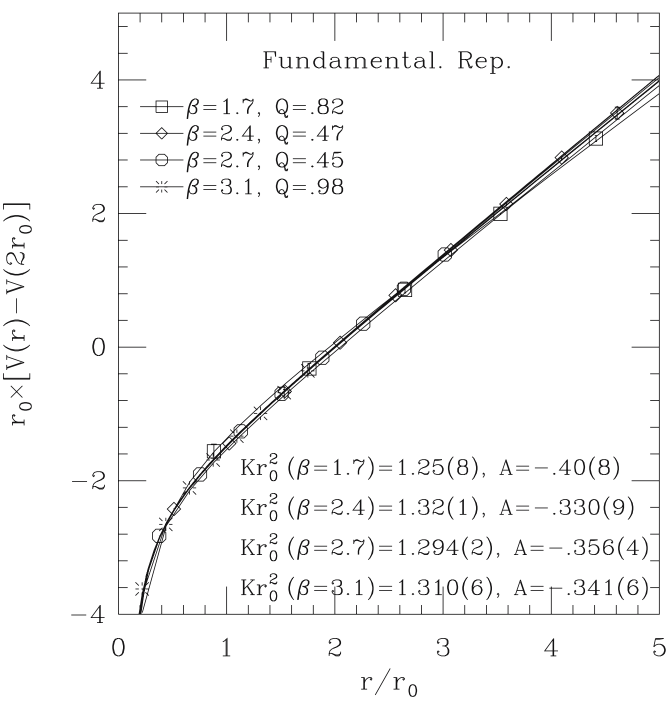

Fig. 3. The static quark potential $V(r)$ in terms of the hadronic scale r_0 for the fundamental representation. The potentials of all four measurements are in agreement. Q represents the confidence level of the fit

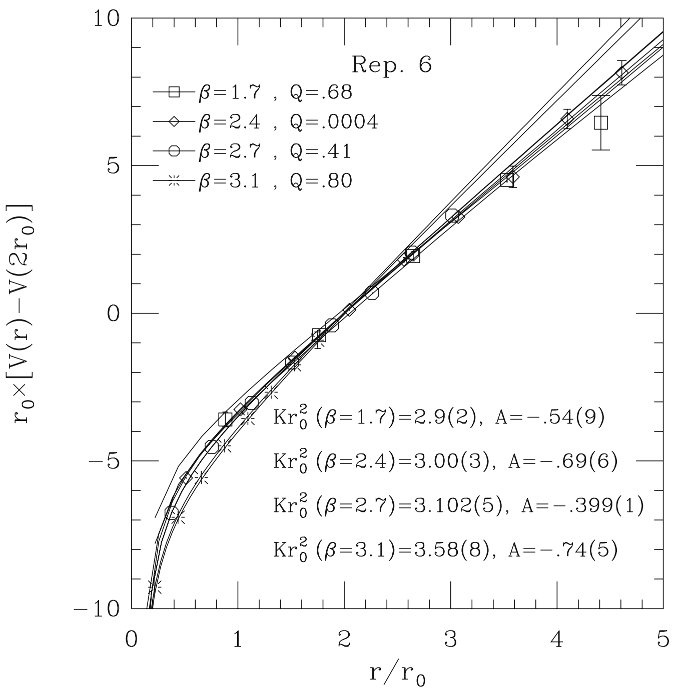

Fig. 4. Same as Fig. 3 but now for representation 6. Results from three lattices, including the new one, scale well. Potentials of $\beta = 3.1$ do not scale since the lattice spatial volume is significantly smaller than the other ones and finite size error affects that measurement

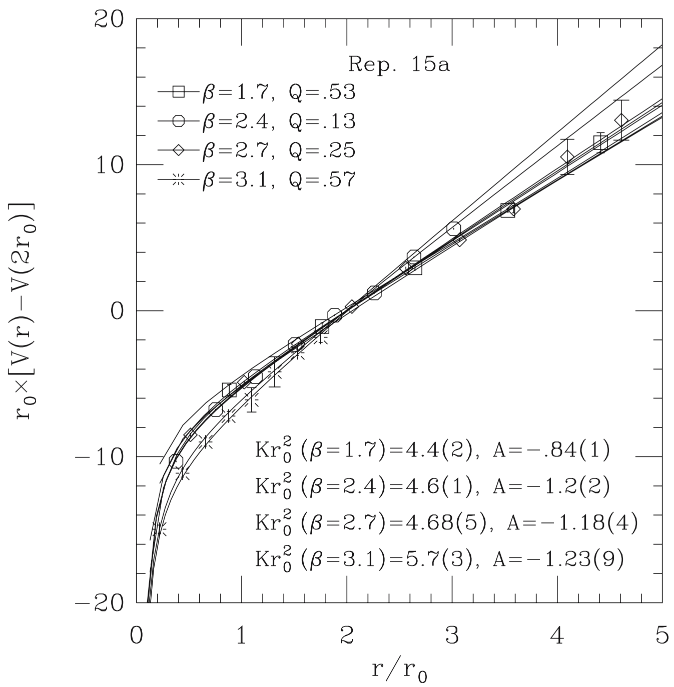

Fig. 5. Same as Fig. 4 but now for representation 15a

the potentials of the finest lattice do not show good scaling behavior. Probably this happens because of the smaller lattice volume of this coupling. Table 1 shows the lattice parameters including lattice volumes and lattice spacings for the four lattices of this study. The volume of the finest lattice, $a_s = 0.11$ fm, is significantly smaller than the others. Therefore if one does not want to encounter the error of the finite volume effect one should use a larger lattice volume. This may happen by increasing the number of lattice points in each dimension or increasing the lattice spacing. The former increases the running time and is not economic. Thus the second option has been chosen and $\beta = 2.7$ is selected, which gives the larger lattice spacing, 0.19 fm, to be compared to the previous finer lattice, where the lattice spacing was equal to 0.11 fm. It is still finer than the two other ones with lattice spacings 0.45 fm and 0.25 fm. I recall that these are spatial lattice spacings, and temporal lattice spacing has been kept the same by choosing appropriate aspect ratios. Potentials from this new measurement scale with the two previous measurements.

Using this new coupling constant and the two previous ones, $\beta = 1.7$ and $\beta = 2.4$, the potentials between static sources are studied and the parameters of the potentials are obtained. Table 2 shows the string tensions from the three scaled measurements. Kr_0^2, the dimensionless string tension of each lattice measurement, and the best estimate for each representation are indicated. The best estimate is found by the weighted average of the three lattice measurements. The first error in the best string tension is the statistical error (from the weighted average), and the second one is the systematic error of discretization determined by the standard deviation. The ratio of the string tension is proportional to the ratio of the Casimir scaling of the 7th column. Table 3 shows the coefficient of the Coulombic term obtained from each lattice measurement. Again the ratio of the coefficient of each representation to that of the fundamental representation is brought for comparison with the ratio of the Casimir scaling.

6 Discussion of string tensions

Lattice calculations [1, 2] show that the potentials between the static $SU(3)$ sources are linear and proportional to the Casimir operator of each representation, which means proportional to the eigenvalue of the quadratic Casimir operator of that representation. The proportionality of the potentials with the Casimir operator is expected at short distances, where the force between heavy quarks may be described by one gluon exchange. But this behavior has not been understood for intermediate distances even though it is observed not only for $SU(3)$ but also for all $SU(N)$ gauge groups (examples are [3, 13] and references therein). Another scenario which tries to explain the linear potentials at intermediate distances is flux tube counting. The idea is that the string tension of higher representation sources can be obtained by multiplying the number of fundamental strings by the string tension of the quarks in the fundamental representation [5]. A fundamental string is a string which connects a fundamental heavy quark to an antiquark. The last column of Table 2 shows the number of fundamental fluxes of each representation. Figure 6 shows the data of a lattice calculation and the thick cen-

Table 1. Lattice parameters for the four lattice measurements. β is the coupling constant; a_s indicates the spatial lattice spacing and ξ shows the ratio of spatial spacing to the temporal one. The number of configurations is shown in the last column. The spatial lattice volume for the finer lattice is significantly smaller than the other ones. Since possibly the finite volume effect error affects this measurement, it has been excluded from further calculations

Lattice	β	$\xi = \frac{a_s}{a_t}$	a_s (fm)	Spatial volume (fm^3)	No configurations
$10^3 \times 24$	1.7	5.0	0.43	79.5	22 400
$18^3 \times 24$	2.4	3.0	0.25	91.1	21 620
$16^3 \times 24$	2.7	2.0	0.19	28.1	12 800
$16^3 \times 24$	3.1	1.5	0.11	5.5	18 200

Table 2. String tensions in terms of r_0 for different coupling constants, lattice sizes, and the best estimate. The ratio of the string tension of each representation to that of the fundamental one is roughly qualitatively in agreement with the ratios of the corresponding Casimir numbers as well as the number of fluxes

Rep.	$Kr_0^2(\beta=1.7)$	$Kr_0^2(\beta=2.4)$	$Kr_0^2(\beta=2.7)$	Best estimate	$\frac{k_r}{k_f}$	$\frac{C_r}{C_f}$	Flux no.
3	1.25(8)	1.32(1)	1.294(2)	1.295(2)(36)	1	1	1
8	2.60(1)	2.60(3)	2.88(2)	2.65(1)(17)	2.05(1)(14)	2.25	2
6	2.9(2)	3.00(3)	3.102(5)	3.10(1)(16)	2.39(1)(14)	2.5	2
15a	4.4(2)	4.6(1)	4.68(5)	4.65(4)(18)	3.59(3)(17)	4.0	3
10	4.9(3)	5.4(2)	5.35(2)	5.35(2)(32)	4.13(2)(27)	4.5	3
27	5.9(5)	6.62(6)	7.48(3)	7.3(1)(10)	5.64(2)(79)	6	4
15s	7.1(5)	7.6(2)	8.1(1)	7.97(9)(67)	6.15(7)(54)	7	4

Table 3. Coulombic coefficients found by different lattice calculations and the best estimate. Rough agreement with the Casimir ratios is observed

Rep.	$A(\beta=1.7)$ $10^3 \times 24$	$A(\beta=2.4)$ $18^3 \times 24$	$A(\beta=2.7)$ $16^3 \times 24$	Best estimate	$\frac{A_r}{A_f}$	$\frac{C_r}{C_f}$
3	−0.40(8)	−0.330(9)	−0.356(4)	−0.352(4)(37)	1	1
8	−0.60(5)	−0.93(3)	−0.69(1)	−0.71(1)(18)	2.02(3)(54)	2.25
6	−0.54(9)	−0.69(6)	−0.798(2)	−0.80(1)(20)	2.27(3)(61)	2.5
15a	−0.84(1)	−1.2(2)	−1.18(4)	−0.86(2)(33)	2.44(6)(97)	4.0
10	−0.50(2)	−0.5(2)	−1.43(1)	−1.24(1)(76)	3.5(1)(22)	4.5
27	−1.9(5)	−1.71(6)	−1.82(1)	−1.82(1)(96)	5.1(1)(28)	6
15s	−1.6(4)	−2.1(2)	−2.28(4)	−2.27(4)(49)	6.5(1)(16)	7

ter vortices model [14]. The thick center vorices model is one of the phenomenological models which tries to describe the behavior of the linear part of the static sources potentials. Cross signs show the string ratios obtained from the lattice calculation of this paper. The Casimir ratios and the number of fundamental flux tubes are indicated by circles and diamonds, respectively. Square signs represent the string tensions ratios obtained from the thick center vortices model. As it is observed, lattice calculations and thick center vortices results agree qualitatively with both Casimir scaling and flux tube counting. The possible reasoning that the string tension is larger than the number of fluxes is discussed in the second reference of [5] and also in [14]. This might happen because at intermediate distances, the fundamental fluxes overlap and a positive energy is added to the binding energy of fluxes and makes the string tension larger than the number of fluxes times the fundamental string tension.

Since the error due to the hadronic scale uncertainty is not considered in our lattice data, the lattice errors are larger than what is reported in the figure. In addition, the potential is supposed to be measured from the area fall-off at large t from the equation $W(r,t) \simeq \exp^{-V(r)t}$. Since for large r values – especially for higher representations – the Wilson loops get too small for large t, and the error due to statistical fluctuations makes the measurements meaningless, a calculation of the potentials using smaller t's is essential. Even though a systematic error by changing the fit range or by comparing with V of smaller r's is obtained for potential of each representation, it seems that the string

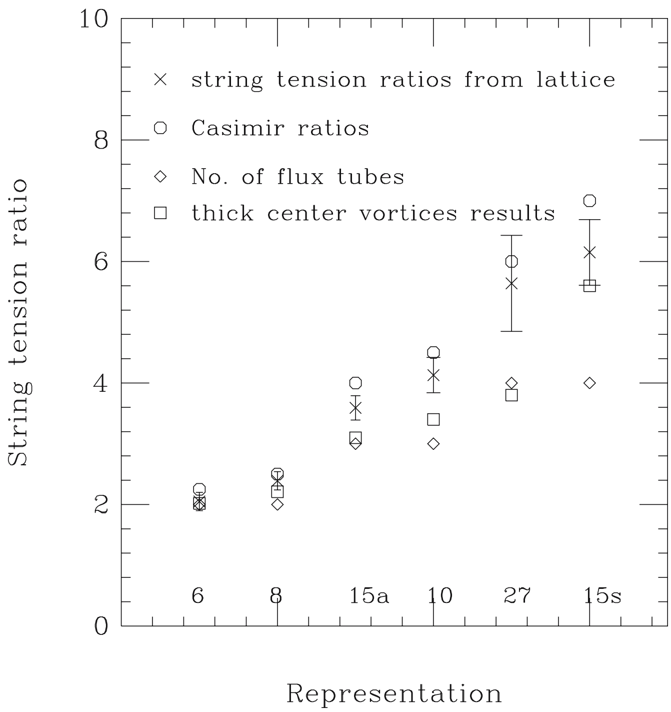

Fig. 6. Ratios of string tensions of $SU(3)$ quarks of different representations to the string tension of quarks in the fundamental representation are plotted. Considering the lattice data errors, a rough agreement with both Casimir scaling and flux tube counting is observed. String ratios obtained from thick center vortices are also shown for comparison

tension is overestimated expecting a larger error especially for higher representations. Figure 1 shows the potential between fundamental quarks versus t for $r = 3$, where r indicates the lattice distance. The fitting range is shown by the solid line. As mentioned before, for higher representations, especially for large r, large t's could not be used. Looking at this plot, it seems that one overestimates the potentials if one measures the potentials using small t's.

7 Conclusion

Using a new coupling constant, the $SU(3)$ potentials between static sources for a variety of representations are obtained. String tensions are found using this new measurement and the author's previous calculations, which have shown good scaling behavior. The data for the finest lattice have been excluded since by comparison, it is observed that the finite volume effect destroys the measurement.

String tensions still remain qualitatively in agreement with both the Casimir scaling and flux tube counting. In fact, the errors of the author's lattice data of this study are still too large to discriminate between the two hypotheses. The results of this study is in agreement with the data of [2] which has found that the potentials are proportional to the Casimir scaling but, however, clearly exclude flux tube counting. Furthermore, since the Wilson loops do not couple well with screened representation, an investigation of the k-string picture for large distances or stable strings could not be done.

The author would like to emphasize that even though there is some evidence of proportionality of the string tensions with the Casimir scaling, at intermediate distances, for $SU(N)$ gauge groups, the Casimir scaling is still a puzzle. Understanding the physics of string tensions and confinement is still an open and interesting subject.

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